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LETTER TO THE EDITOR

Illustration of the causal model of quantum statistics

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Abstract. As an illustration of the causal stochastic interpretation of quantum statistics it is shown, in the case of two particles in two harmonic oscillator wavepackets, that the different results for observables obtained with different statistics can be explained in terms of distinguishable particle motions correlated by the quantum potential.

In two recent letters (Kyprianidis *et al* 1984, Cufaro-Petroni *et al* 1984), following the line introduced by Tersoff and Bayer (1983) it was shown that quantum statistics result from the motion of distinguishable particles correlated by causal non-local actions-at-a-distance. The difference between classical Maxwell–Boltzmann (MB) and quantum Bose–Einstein (BE) or Fermi–Dirac (FD) statistics was shown to result from the uncontrollable non-local character of stochastic interactions connecting particles embedded in Dirac's random aether for the quantum case. Furthermore it was argued that FD statistics can be distinguished from BE statistics by the fact that identical half-integer spin particles induce repulsive gauge fields in the case of parallel spin alignment. This preserves the $n_i = 0$ or 1 occupation numbers of phase-space cells.

In the present letter we wish to demonstrate explicitly in a typical physical situation how the individual motions of particles under the influence of the many-body quantum potential lead to different statistical results according to the type of wavefunction assumed. The causal interpretation of quantum statistics (Bohm and Hiley 1975, Vigier 1982, Guerra and Moratov 1983) is thus shown to provide an intuitive understanding of quantum statistical results in terms of correlated particle motions, classical statistics arising as a special case when the particles are not correlated by the quantum potential. The case examined here is the following. Consider a harmonic oscillator potential $V = kx^2/2 = mw^2x^2/2$ and construct by solving the Schrödinger equation a wavepacket solution (Bohm 1951)

$$\psi(x, t) = \exp(-i\omega t) \exp[-\tfrac{1}{2}(x - x_0 \cos \omega t)^2] \times \exp[\tfrac{1}{2}i(\tfrac{1}{2}x_0^2 \sin 2\omega t - 2xx_0 \sin \omega t)]. \quad (1)$$

This wavepacket solution is non-dispersive and depending on the time parameter t defines in the causal interpretation a set of possible trajectories for a particle located at the position x, where x_0 is the centre of a wavepacket.

Now consider the case of two particles, one in each of the wavepackets $\psi_A(x_1, t)$ and $\psi_B(x_2, t)$ in the harmonic oscillator potential. The packet $\psi_A(x_1, t)$ is assumed to be centred at x_0 and, in order to simplify the calculations, the packet $\psi_B(x_2, t)$ centred at $-x_0$.

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It is clear that depending on the assumed statistics (MB, BE or FD) three wavefunctions can be written. These are:

$$\phi_{MB} = \alpha_{MB}\psi_A(x_1, t)\psi_B(x_2, t), \tag{2}$$

$$\phi_{BE} = \alpha_{BE}[\psi_A(x_1, t)\psi_B(x_2, t) + \psi_B(x_1, t)\psi_A(x_2, t)], \tag{3}$$

$$\phi_{FD} = \alpha_{FD}[\psi_A(x_1, t)\psi_B(x_2, t) - \psi_B(x_1, t)\psi_A(x_2, t)], \tag{4}$$

where the α's are normalisation constants to be determined by the condition $\iint \phi \, dx_1 \, dx_2 = 1$. This yields for the ψ's of the form of equation (1) the following results:

$$\alpha_{MB} = (\pi)^{-1/2}, \qquad \alpha_{BE} = [2\pi(1 + e^{-2x_0^2})]^{-1/2}, \qquad \alpha_{FD} = [2\pi(1 - e^{-2x_0^2})]^{-1/2}. \tag{5}$$

A standard quantum mechanical calculation yields the mean squared separation of the particles by evaluating

$$\langle(x_1 - x_2)^2\rangle = \int\int_{-\infty}^{\infty} \phi^*(x_1 - x_2)^2\phi \, dx_1 \, dx_2$$

giving the following three results depending on the statistics obeyed by the particles,

$$\langle(x_1 - x_2)^2\rangle_{MB} = 1 + 4x_0^2 \cos^2 wt,$$

$$\langle(x_1 - x_2)^2\rangle_{BE} = 1 + 4x_0^2 \cos^2 wt - e^{-2x_0^2} 4x_0^2/(1 + e^{-2x_0^2}),$$

$$\langle(x_1 - x_2)^2\rangle_{FD} = 1 + 4x_0^2 \cos^2 wt + e^{-2x_0^2} 4x_0^2/(1 + e^{-2x_0^2}), \tag{6}$$

or, by performing a time averaging $\overline{\cos^2 \omega t} = \frac{1}{2}$,

$$\langle(x_1 - x_2)^2\rangle_{MB} = 1 + 2x_0^2,$$

$$\langle(x_1 - x_2)^2\rangle_{BE} = 1 + 2x_0^2 - e^{-2x_0^2} 4x_0^2/(1 + e^{-2x_0^2}),$$

$$\langle(x_1 - x_2)^2\rangle_{FD} = 1 + 2x_0^2 + e^{-2x_0^2} 4x_0^2/(1 + e^{-2x_0^2}).$$

We clearly see from this that the mean squared separation of the particles is decreased in the BE case and increased in the FD case with respect to the MB case.

The explanation given for these predictions in the purely probabilistic Copenhagen interpretation of quantum statistics is evidently:

(i) for MB statistics ϕ_{MB} is factorisable so we are dealing with independent particles;

(ii) for BE and FD statistics when the probability packets overlap in space–time then one cannot tell to which packet the particles belong. We have a correlation due to this uncertainty.

At this stage we claim that these results can be interpreted in a much more satisfactory way in the frame of the SIQM, where the particles, even if they are independent, are thought of as being tied by a permanent non-local action-at-a-distance described by the quantum potential

$$Q = -(\hbar^2/2m)(\Delta_1 R/R + \Delta_2 R/R)$$

are submitted to the quantum forces $F_Q^1 = \nabla_1 Q$ and $F_Q^2 = \nabla_2 Q$ and follow trajectories given by $p_1 = \nabla_1 S$, $p_2 = \nabla_2 S$ where the wave fields are assumed to have the form $\phi = R\, e^{is}/\hbar$. If we apply this scheme to the above problem then we find in the MB case, the quantum potential

$$Q_{MB} = -(\hbar^2/2m)[(x_1 - x_0 \cos \omega t)^2 + (x_2 + x_0 \cos \omega t)^2 - 2]$$

which is composed of two additive contributions from each particle separately. There the quantum forces acting on each particle

$$F_Q^1 = \partial Q/\partial x_1 = -(\hbar^2/m)(x_1 - x_0 \cos \omega t),$$
$$F_Q^2 = \partial Q/\partial x_2 = -(\hbar^2/m)(x_2 + x_0 \cos \omega t)$$

depend only on the particle's coordinate and can be shown after performing an averaging process

$$\langle F_Q^{1,2}\rangle_{\mathrm{av}} = \int\!\!\int_{-\infty}^{\infty} \phi_{\mathrm{MB}}^* F_Q^{1,2} \phi_{\mathrm{MB}} \, \mathrm{d}x_1 \, \mathrm{d}x_2$$

and a time average to vanish identically. Thus we see that the two particles (in the wavepackets) perform completely independent motions and do not interfere.

In the case of BE or FD statistics a lengthy but straightforward calculation yields the following basic results.

(a) The quantum potential Q_{FD} or Q_{BE} cannot be reduced to a form $Q = Q_1(x_1) + Q_2(x_2)$ but has the general form $Q(x_1, x_2)$ which establishes the real physical correlation between the two particles.

(b) The quantum forces $(F_Q^{\mathrm{FD}})_{1,2}$ and $(F_Q^{\mathrm{BE}})_{1,2}$ do depend on the coordinates of both particles and do not average to zero. The trajectory of each particle depends on that of the other and the quantum potential is such that on average the Bose particles approach each other more closely in the region of interference or overlap of the wavepackets than do the Fermi particles. This is a result of the fact that the two contributions to the total wavefunctions interfere with different signs in the two cases leading to different quantum potentials, with an interchange of maxima and minima.

This can be seen by calculating trajectories through numerical integration of the equation $p_i = m\, \mathrm{d}x_i/\mathrm{d}t = \nabla_i S$ with suitable choice of initial particle positions. A numerical integration of this relation gives the particle's trajectories $x_1(t)$ and $x_2(t)$ in the harmonic oscillator potential which are represented in figure 1.

This figure provides us with the basic physical features of the process. The MB particles, being independent, possess trajectories that cross one another. They propagate undisturbed and produce no interference. This is not the case for BE or FD particles. They do not cross but from interference patterns in which the two particles are on the average closer together in the BE case than in the FD case. In the limiting case where the two wavepackets coincide we have

$$\langle (x_1 - x_2)^2\rangle_{\mathrm{MB}} = \langle (x_1 - x_2)^2\rangle_{\mathrm{BE}},$$

the two statistics are equivalent and no interference arises in the BE case. $\langle (x_1 - x_2)^2\rangle_{\mathrm{FD}}$ no longer has a meaning since $\phi_{\mathrm{FD}} = 0$. If the two wavepackets are initially far apart i.e. x_0 large, then all three statistics tend to yield the same result i.e.

$$\langle (x_1 - x_2)^2\rangle = 1 + 4x_0^2 \cos^2 \omega t$$

because the period during which the packets overlap is negligible and the interference of the wavepackets, which yields the different statistics and gives rise to the third term of equation (6), becomes infinitely small. We find that the trajectories of the particles are the same for each statistic outside the region of overlap; the differences arise within it.

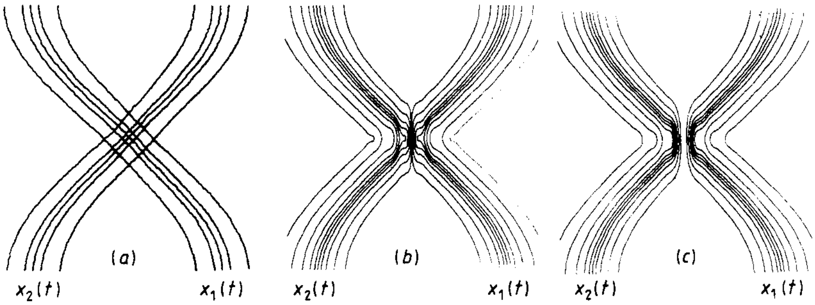

Figure 1. An ensemble of two particle trajectories $x_1(t)$, $x_2(t)$ with initial positions such that $x_2(0) = -x_1(0)$ and a concentration of particle trajectories around the packet maxima. (*a*) Maxwell–Boltzman (*b*) Bose–Einstein (*c*) Fermi–Dirac.

The correlation effects mediated by the quantum potential between the two particles determines their physical behaviour and conditions their different statistical averages of physical variables or observables. This can be easily understood in the SIQM where particles obeying quantum statistics are constantly submitted to the stochastic random motions of the underlying subquantal medium, the Dirac aether (Dirac 1951, 1952). The symmetric or antisymmetric character of the system's wavefunction is a consequence of the existence (or not) of local repulsive gauge fields and not a first quantum mechanical principle. Finally, the factorisability of the wavefunction for MB statistics can be thought of as a consequence of the existence of definite phase relations, such that the particle's motions are in fact independent, which will be studied in a subsequent publication. The quantum potential is a useful concept that illustrates the existence of the action-at-a-distance correlations of the underlying random chaotic Dirac aether and provides an understanding of all quantum phenomena in terms of real motions in physical space–time.

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